

Bi-Jurisdictional Monte Carlo Retirement Withdrawal Optimizer

Methodology, Tax Engine, and Statutory Reference

Scope	United States (federal + state) and Canada (federal + provincial)
Tax model precision	2025 brackets, basis-point validated against CRA and IRS published rates
Simulation	Monte Carlo (default 2,000 sims × user-defined horizon)
Objective function	Minimize lifetime personal tax; success criterion: terminal-year wealth ≥ 0
Bucket coverage (US)	Taxable, Traditional 401(k)/IRA, Roth, HSA, C-Corporation
Bucket coverage (CA)	Non-registered, RRSP/RRIF, TFSA, FHSA, CCPC Holdco
Mandatory distributions	IRS Uniform Lifetime Table (RMD age 73+); CRA Prescribed Factor Table (RRIF age 72+)
Dividend modeling	Eligible/non-eligible split (CA); qualified dividend channel (US)
Leverage modeling	Smith Manoeuvre (personal); CCPC corporate borrowing; both with ITA s.20(1)(c) interest deduction
Output formats	Interactive HTML dashboard with sliders; Safe Withdrawal Rate frontier across confidence levels

Statutory references throughout: Internal Revenue Code (I.R.C.), Treasury Regulations, Income Tax Act (ITA), Canada Revenue Agency (CRA) Income Tax Regulations, Ontario Income Tax Act, and case law (Singleton, Lipson, MacDonald, Foix, Aspro). All values reflect 2025 tax tables.

1. Executive Summary

Retirement withdrawal planning under uncertainty is one of the more analytically complex problems faced by households at the boundary between accumulation and decumulation. The retiree must simultaneously model: (a) stochastic portfolio returns over a 20-40 year horizon, (b) progressive income-tax brackets that interact differently with different account types, (c) mandatory distribution rules that force taxable income above what would be optimal, (d) means-tested government benefits that claw back at higher incomes, and (e) the differential tax treatment of dividends, capital gains, and ordinary income across multiple jurisdictions.

This tool implements a Monte Carlo simulation framework that handles all of the above for both United States and Canadian retirees, with statutory-grade tax engines validated against published rates to basis-point precision. The objective function defaults to minimizing total lifetime personal tax (equivalently, maximizing after-tax wealth available for spending and bequest) subject to a user-specified spending need and a probabilistic success constraint.

Key methodological contributions

- Iterative bracket-fill withdrawal cascade with combined federal-plus-state-or-provincial marginal-rate awareness, automatically avoiding needless triggering of OAS recovery tax, NIIT, or surtax bands.
- Full Canadian dividend taxation including eligible (39.34% top Ontario) and non-eligible (47.74% top Ontario) channels, both with statutory gross-up, federal DTC, and provincial DTC computed after surtax per ITA(ON) s.21.
- Canadian Holdco / CCPC modeling with corrected dividend-only inside drag: capital appreciation compounds tax-free inside the corp until realization, then converts to capital gain via section 70(5) deemed disposition and section 84.1-compliant pipeline (~26.77% effective Ontario).
- US C-Corporation modeling with three extraction strategies (W-2 salary, qualified dividend, defer-to-death with I.R.C. section 1014 basis step-up).
- Smith Manoeuvre and corporate-leverage modeling per ITA s.20(1)(c): interest expense deductible against investment income, with proper sequence-of-returns and tail-risk amplification visible in the wealth-fan distribution.
- Mandatory distributions per the IRS Uniform Lifetime Table (Treasury Reg section 1.401(a)(9)-9, post-SECURE 2.0 section 107) and CRA Prescribed Factor Table (Income Tax Regulations s.7308(4)).
- Safe Withdrawal Rate frontier solved via bracket-and-binary search across confidence levels of 50%, 70%, 80%, 85%, 90%, 95%, and 99%.
- Cross-country bucket equivalence mapping that preserves user-entered values across the US/CA toggle (Taxable to Non-reg, Trad to RRSP, Roth to TFSA, HSA to FHSA, C-Corp to Holdco, Social Security to CPP).

2. Monte Carlo Simulation Framework

Each simulation path consists of an annual loop from current age to end age. Within each year:

- **Random return draw:** $r \sim \text{Normal}(\mu, \sigma^2)$ in real (post-inflation) terms, applied uniformly to all buckets except those with corporate-tax drag.
- **Corporate-bucket adjustment:** Canadian Holdco grows at $r - d \times \text{drag}$ where d is the dividend yield and drag the user-set corp tax rate on the dividend portion (default 30%, representing US dividends in a CCPC). The capital appreciation portion ($r - d$) compounds tax-free inside the corp until realization. US C-Corp in dividend/defer modes grows at $r \times (1 - \text{drag})$ (US C-corps have no preferential cap-gain treatment).
- **Loan interest expense** (if leverage active): paid out of the leveraged bucket(s) as a cash outflow proportional to each bucket's share of the loan. Loan balance is constant (interest-only).
- **Mandatory distributions collected:** RMD from Traditional 401(k)/IRA at age 73+ per the IRS Uniform Lifetime Table; RRIF withdrawals from RRSP at age 72+ per CRA s.7308(4). Government pensions (Social Security, CPP, OAS) collected at user-specified ages.
- **Withdrawal cascade for additional spending:** iteratively gross-up from the registered/tax-deferred bucket until the chosen marginal-rate ceiling, then draw from non-registered (with capital-gain proration), then from corporate buckets per chosen strategy, then from Roth/TFSA, then from HSA/FHSA.
- **Tax computation:** federal + state/provincial layers with all jurisdictional features (SS provisional-income method, NIIT, FICA, surtax, OAS recovery, BPA, age amount, DTCs).
- **Shortfall accounting:** any unmet spending need accumulates in a deficit tracker that compounds at the portfolio rate; trajectories are tracked as bucket-sum minus deficit (can go negative legitimately).
- **Success criterion:** terminal-year wealth ≥ 0 (does the estate end solvent). This includes corporate-bucket pipeline-tax adjustment at terminal ($\text{Holdco} \times (1 - \text{pipeline rate})$) and loan repayment.

Algorithmic complexity

For N simulations and Y years, total operations are $O(N \times Y \times K)$ where K is the per-year work (bracket-fill iterations times tax-engine calls; typically K is approximately 20). Default parameters ($N=2000$, $Y=25$) complete in under 3 seconds in a modern browser. The Safe Withdrawal Rate frontier requires additional $N \times Y$ operations per binary-search step per confidence level (7 levels \times ~ 10 steps \times 1500 sims, approximately 110x the work of one primary MC; ~ 30 seconds typical).

3. Tax Engine - United States

Federal layer

Component	Implementation	Statutory basis
Brackets	2025 MFJ / Single / MFS / HoH per IRS Rev. Proc. 2024-40	I.R.C. s.1(j)
Standard deduction	\$15,000 single / \$30,000 MFJ + \$1,600 age-65 bump each	I.R.C. s.63(c)
LTCG / Qualified dividend	0% / 15% / 20% stacked above ordinary income	I.R.C. s.1(h), s.1(h)(11)
NIIT	3.8% on min(NII, MAGI - threshold); thresholds \$200k single / \$250k MFJ	I.R.C. s.1411
FICA (salary mode)	6.2% OASDI to wage base \$176,100 + 1.45% Medicare uncapped + 0.9% Additional Medicare above MAGI thresholds	I.R.C. s.3101
SS taxation	Provisional-income method up to 85% taxable	I.R.C. s.86
RMD start age	73 (per SECURE Act 2.0 s.107; moves to 75 in 2033)	I.R.C. s.401(a)(9)
RMD divisor	Uniform Lifetime Table, extended to age 120	Treasury Reg s.1.401(a)(9)-9

State layer

Five states modeled with full bracket detail for 2025: California (top 13.3% with mental-health surcharge), New York (top 10.9%), Massachusetts (5% flat plus 4% millionaire surtax), Illinois (4.95% flat), and New Jersey (top 10.75%). LTCG and qualified dividends are taxed at the ordinary state rate since no US state has a meaningful preferential treatment (Massachusetts taxes short-term gains at 8.5% but long-term at 5%; that nuance is captured). Other states default to a 0% flat (user can override).

Bucket types and treatment

Bucket	Tax during life	Tax at withdrawal / death
Taxable brokerage	Annual qualified dividend tax (LTCG-rate + NIIT)	Capital gain at sale: 50% inclusion times marginal; gain = sale - basis
Trad 401(k) / IRA / 403(b) / 457(b) / SEP / SIMPLE	Tax-deferred growth	Full withdrawal taxed as ordinary income; RMD from age 73
Roth IRA / 401(k)	Tax-free growth	Tax-free withdrawal; no RMD per I.R.C. s.408A(c)(5); Roth 401(k) RMD eliminated 2024 per SECURE 2.0 s.325

Bucket	Tax during life	Tax at withdrawal / death
HSA	Tax-free growth	Tax-free for qualified medical; otherwise ordinary income
C-Corporation (passive)	Inside drag (~25% default; salary mode = 0% drag since salary deducts)	Three modes: W-2 salary (ordinary + FICA), qualified dividend (LTCG + NIIT + state), defer (s.1014 basis step-up at death)

4. Tax Engine - Canada

Federal layer

Component	Implementation	Statutory basis
Brackets	2025 federal brackets (15%, 20.5%, 26%, 29%, 33%)	ITA s.117
Basic Personal Amount	\$15,705 (2025)	ITA s.118(1)
Age amount (65+)	\$8,790 with clawback above \$44,325	ITA s.118(2)
Capital gains	50% inclusion of realized gain	ITA s.38(a)
Eligible dividend gross-up	1.38x with federal DTC 15.0198% of grossed-up	ITA s.82(1)(b), s.121
Non-eligible dividend gross-up	1.15x with federal DTC 9.0301% of grossed-up	ITA s.82(1)(a), s.121
OAS recovery (clawback)	15% of income above \$90,997 (2025)	ITA s.180.2
RRIF start age	72 (year after age-71 RRSP conversion deadline)	ITA s.146.3(1)(b.1)
RRIF prescribed factor	CRA published table, ages 72-95+ (s.7308(4)); pre-71 formula $1/(90 - \text{age})$ per s.7308(3)	Income Tax Regs s.7308

Provincial layer

Province	Top combined ord	Top elig div	Top cap gain	Special features
Ontario (ON)	53.53%	39.34%	26.77%	Surtax 20% + 36% on basic prov tax above tier thresholds
British Columbia (BC)	53.50%	36.54%	26.75%	Top bracket above \$252,752
Alberta (AB)	48.00%	34.31%	24.00%	Flat 10% provincial above \$148,269
Quebec (QC)	53.31%	40.11%	26.65%	Federal abatement 16.5%; QPP instead of CPP

Bucket types and treatment

Bucket	Tax during life	Tax at withdrawal / death
Non-registered	Annual dividend tax (eligible 1.38x gross-up / non-elig 1.15x gross-up; both with DTCs)	Cap gain on sale: 50% inclusion x marginal rate
RRSP / RRIF	Tax-deferred growth	Full withdrawal as ordinary income; RRIF minimum from age 72
TFSA	Tax-free growth	Tax-free withdrawal; no minimum
FHSA	Tax-free growth (qualifying)	Tax-free for qualifying home purchase; otherwise ordinary income
Holdco (CCPC passive)	Dividend-only inside drag (~30% default for US div mix; ~0% for Canadian eligible div via s.112 deduction). Unrealized capital appreciation tax-free until disposition.	Two modes: Pipeline (defer-to-death, ~26.77% Ontario via s.70(5) + s.84.1) or annual non-eligible dividend (~47.74% top Ontario)

5. Withdrawal Cascade and Bracket-Fill Optimization

Each year the simulator must convert spending need (after-tax cash required) into specific withdrawal amounts from each bucket. The cascade implements a tax-optimal sequence based on user-selected withdrawal preference:

Tax-optimized auto-fill (default): bracket-fill the tax-deferred bucket up to a marginal-rate ceiling, then top up from non-registered (with cost-basis-aware capital-gain proration), then from corporate buckets per the chosen extraction strategy, then from tax-free buckets (Roth/TFSA), and finally from HSA/FHSA.

Cascade order

Stage	US bucket order	CA bucket order
1. Mandatory income	RMD (73+), Social Security	RRIF minimum (72+), CPP, OAS
2. Bracket-fill from tax-deferred	Traditional 401(k)/IRA up to bracket ceiling	RRSP up to bracket ceiling
3. Taxable / Non-reg	Capital-gain-aware draw against basis	Capital-gain-aware draw against basis
4. Corporate bucket	C-Corp (salary / dividend / defer)	Holdco (pipeline / dividend)
5. Tax-free	Roth IRA / 401(k)	TFSA
6. HSA / FHSA	HSA (tax-free for medical)	FHSA

Marginal-rate awareness

The bracket-fill loop computes the combined federal + state/provincial marginal rate at the current income level and stops the Traditional/RRSP draw when that rate would jump to the next bracket above the user-selected ceiling. For Canadian profiles the computation also includes OAS clawback (15% added to effective rate when income is in the \$90,997-\$148,997 range), Ontario surtax (20% on basic prov tax above \$5,710 + additional 36% above \$7,307, fully implemented), and the age-amount clawback (\$8,790 × 15% above \$44,325).

For US profiles the computation includes the 3.8% NIIT when MAGI is above the threshold for investment income, and the 0.9% Additional Medicare for salary-mode C-Corp extractions.

This means the engine actively avoids needless triggering of these phase-outs and surcharges when alternative withdrawals are available - for example, in a year where pulling another \$10k from the RRSP would push the retiree into the OAS clawback band, the engine will instead pull that \$10k from the TFSA or Non-reg.

6. Mandatory Distributions

Both jurisdictions require minimum annual withdrawals from tax-deferred accounts starting at a specified age. The model uses the official published tables to basis-point precision.

IRS Uniform Lifetime Table (sample)

Treasury Reg s.1.401(a)(9)-9, as amended effective 2022. Applies to Traditional 401(k) / 403(b) / 457(b) / Traditional IRA / SEP / SIMPLE. RMD start age 73 per SECURE Act 2.0 s.107 (moves to 75 in 2033). Annual RMD = prior-year-end balance / divisor. Roth IRA: no RMD per I.R.C. s.408A(c)(5). Roth 401(k): RMD eliminated 2024 per SECURE 2.0 s.325. HSA: no RMD.

Age	Divisor	%/yr	Age	Divisor	%/yr
73	26.5	3.77%	95	8.9	11.24%
75	24.6	4.07%	100	6.4	15.63%
80	20.2	4.95%	105	4.6	21.74%
85	16.0	6.25%	110	3.5	28.57%
90	12.2	8.20%	120	2.0	50.00%

CRA Prescribed Factor Table (sample)

Income Tax Regulations s.7308(4), post-2015 schedule. RRSP must convert to RRIF (or annuity) by December 31 of the year the holder turns 71. First mandatory minimum is the year after conversion (age 72), using the age-72 prescribed factor of 5.40%. For voluntary early conversion (ages 65-70), the formula $1/(90 - \text{age})$ applies per s.7308(3). Holder may elect to use spouse's (younger) age for the factor under s.146.3(1)(b.1) (not modeled in this version).

Age	Factor	%/yr	Age	Factor	%/yr
72	0.0540	5.40%	85	0.0851	8.51%
75	0.0582	5.82%	90	0.1192	11.92%
80	0.0682	6.82%	95+	0.2000	20.00%

7. Dividend Yield Modeling

The user-set dividend yield (%) determines what fraction of the Non-registered (CA) or Taxable (US) bucket's total real return is distributed as taxable dividend versus retained as deferred capital appreciation. Dividends are assumed to be fully reinvested (DRIP - bucket value unchanged after distribution) but immediately taxable; the resulting tax is funded via the withdrawal cascade.

Canadian eligible vs non-eligible split

The **Eligible Canadian div %** input splits the Non-reg dividend yield between two distinct tax channels. Eligible dividends (publicly-traded Canadian corp distributions, most Canadian dividend ETFs like XIU/XDV, GRIP-tracked sources) are grossed up 38% with federal DTC 15.0198% and provincial DTC per province (ON 10%, BC 12%, AB 8.12%, QC 11.7%). Non-eligible dividends (CCPC distributions from private corps, some income trusts and split-share corps) are grossed up 15% with federal DTC 9.0301% and provincial DTC (ON 2.9863%). DTCs are applied after surtax per ITA(ON) s.21.

Income type	Top ON effective	Statutory basis
Ordinary income / interest	53.53%	ITA s.117 federal + s.21(2)-(3) Ontario
Non-eligible dividend (CCPC / Holdco)	47.74%	ITA s.82(1)(a), s.121, ON 2.9863% DTC
Eligible Canadian dividend	39.34%	ITA s.82(1)(b), s.121, ON 10% DTC
Foreign / US dividend in Non-reg	~53.5% + 15% US withholding (treaty, partially recoverable via FTC)	Treated as ordinary income
Capital gain (realized, top ON)	26.77%	ITA s.38(a): 50% inclusion times 53.53%

US qualified dividend treatment

US Taxable bucket dividends are treated as qualified per I.R.C. s.1(h)(11) (assumes >60-day holding period in broad-market equity ETFs). Federal tax stacks on the LTCG ladder at 0%/15%/20%; 3.8% NIIT applies above MAGI thresholds; state tax applies at the ordinary rate (no state has a meaningful preferential rate). Bucket basis is held constant - reinvested dividends do not step up basis in the model (a simplification; actual IRS treatment increases basis by the reinvested amount).

8. Holdco and C-Corporation Modeling

Canadian Holdco (CCPC passive investment)

The Holdco bucket models a Canadian Controlled Private Corporation holding passive investments. Inside-corp tax mechanics differ by income type, and crucially **unrealized capital gains compound tax-free until disposition**:

Income type inside corp	Effective rate	Statutory mechanism
US / foreign dividends	~30% net	Part I refundable + 15% US withholding offset by foreign tax credit
Interest income	~50% gross	Part I refundable ~50.17% Ontario; RDTOH-integrated at distribution
Canadian eligible dividends	~0% net	s.112 inter-corp deduction; Part IV refundable 38% recovered via RDTOH
Unrealized capital gains	0%	Deferred per ITA s.38(a) until disposition
Realized capital gains	~25% / ~12.5% with CDA	50% inclusion times ~50% Part I; other 50% to Capital Dividend Account (distributable tax-free)

The model's **Corp tax on dividends only** parameter (default 30%) is applied ONLY to the dividend yield portion of Holdco returns, not to the capital appreciation portion. Effective Holdco growth = $r - d \times drag$. At $\mu=4.5\%$, $d=2\%$, $drag=30\%$, this is $4.5\% - 0.6\% = 3.9\%/yr$ - versus the previous (incorrect) formula $r \times (1 - drag) = 3.15\%/yr$. Over 25 years this is a ~21% terminal value differential.

Extraction strategies (Canadian Holdco)

1. Pipeline - defer to death (default): no withdrawal during life; capital appreciation compounds tax-free inside the corp for the full horizon. At death, ITA s.70(5) deemed disposition triggers a capital gain on Holdco shares (50% times top marginal personal ~26.77% Ontario). Estate executes a post-mortem pipeline per s.84.1-compliant Newco transfer: inherits at stepped-up basis, transfers to Newco for note, Newco extracts cash as inter-corp dividend tax-free under s.112, then repays note as capital. Net rate ~27% versus ~47.7% for annual non-eligible dividend.

2. Annual non-eligible dividend: Holdco distributes during life. Added to personal taxable income at 1.15x gross-up, taxed at marginal rate, less federal and provincial DTCs. Top Ontario effective: 47.74% (matches CRA published).

Antithesis - when pipeline fails: CRA has historically challenged pipelines via ITA s.84(2) deemed-dividend recharacterization (*MacDonald v. Canada*, 2013 FCA 110). The safe structure requires ≥ 1 -year delay between Newco formation and Holdco wind-up, gradual distribution, and legitimate business purpose for Newco. CRA's 2018 Advance Income Tax Rulings standardized acceptable forms but Finance has periodically signaled tightening. GAAR (s.245) remains theoretically applicable; *Foix v. Canada* (2024 TCC) extended scrutiny to certain variants.

US C-Corporation passive holding

The C-Corp bucket models a small private US C-corporation used as a passive investment vehicle. Three extraction modes:

Mode	During life	Terminal handling
W-2 salary (default)	Corp deducts salary, no corp tax on retained earnings, bucket grows at full r. Salary taxed as personal ordinary + 7.65% FICA + 0.9% Additional Medicare above threshold.	Bucket at face value
Qualified dividend	Inside drag (~25% default = federal 21% + state) applied to all returns. Distribution taxed at LTCG rate (0/15/20%) + 3.8% NIIT + state ordinary.	Bucket at face value
Defer to death	No withdrawal; inside drag applies. I.R.C. s.1014 basis step-up assumption - heirs liquidate at zero additional shareholder gain.	Bucket times 1.0 (assumes step-up)

Statutory references: I.R.C. s.1(h)(11) qualified dividend; s.1411 NIIT; s.1014 step-up basis at death; s.3101(b)(2) Additional Medicare; s.11 corporate income tax; s.541 personal holding company tax (potential trap, not modeled); s.531-537 accumulated earnings tax (potential trap, not modeled).

9. Leverage - Smith Manoeuvre and Holdco Borrowing

Canadian users may model leveraged investing per ITA s.20(1)(c), which makes interest on money borrowed to earn investment income fully deductible against income from property. This converts annual dividend taxation into deferred capital-gain taxation - a substantial structural advantage.

Mechanism implemented

- Loan amount (user-specified as % of initial portfolio) injected into the chosen bucket at simulation start with matching basis (no embedded gain on borrowed funds).
- Loan balance constant (interest-only structure typical for HELOCs / investment loans).
- Annual interest expense = loan times user-specified real borrowing rate, paid out of the borrowing bucket as a cash outflow.
- Interest expense deducted from taxable ordinary income (simplification: actual mechanic per s.20(1)(c) deducts from investment income, with excess offsetting other ordinary income or carrying forward 20 years as a non-capital loss per s.111).
- Three location modes: Personal Non-reg (default, full s.20(1)(c) deduction at personal level), Holdco (corporate borrowing, currently deducted at personal level as a simplification), or Both (user-specified % split between Personal and Holdco).
- Terminal wealth subtracts outstanding loan balance - visible in P10/P50/P90 distribution.

Tax conversion math

At top Ontario, dividends previously taxed at 39.34% (eligible) or 47.74% (non-eligible, Holdco) get offset by deductible interest. When loan is sized such that $L \times i \sim (P + L) \times d$, all annual dividend income is wiped out. The full leveraged return then realizes only as **capital gain at 26.77%** when eventually sold or via post-mortem deemed disposition. Breakeven leverage to fully wipe dividends: $L = P \times d / (i - d)$. With $d=2\%$, $i=3\%$: $L = 2P$ (200% leverage zeros total dividend tax). With $d=2\%$, $i=5\%$: $L = 2P/3$ (67% leverage zeros it). When $i \leq d$, dividends self-finance interest and any leverage adds windfall return.

Expected windfall return

The leverage spread ($\mu - i$) compounds annually on the borrowed amount. At $\mu=4.5\%$, $i=3\%$, the spread is 1.5%/yr. At 50% leverage, the additional portfolio return is approximately 0.75%/yr before realization tax. Over 25 years, this gross extra wealth is approximately $P \times leverage_pct \times ((1 + spread)^{years} - 1) \sim P \times 0.50 \times 0.45 = 22.5\%$ of initial portfolio.

Catastrophic-event risks (modeled in the wealth distribution)

Risk	Mechanism	When it bites
Sequence-of-returns amplification	Early-year drawdown shrinks portfolio while loan remains fixed; leverage ratio mechanically rises just when most fragile.	First 5-10 years of retirement

Risk	Mechanism	When it bites
Margin call / forced deleveraging	Lender LTV breach triggers forced liquidation at depressed prices, crystallizing losses. NOT simulated - actual P10 could be worse.	40%+ drawdowns; brokerage loans most prone
Interest-rate spike	Variable-rate loans repriced; if i jumps above $\mu - d$, leverage becomes negative-return.	2022-style rapid hiking cycles
CRA deductibility challenge	Singleton/Lipson 'reasonable expectation of income' test failure if portfolio shifts to zero-yield growth equities.	Composition drift; audit risk
Insolvency cascade	Forced liquidation in major drawdown may not cover loan principal, leading to personal bankruptcy. Recourse on personal Smith Manoeuvre.	$\geq 75\%$ leverage tail
Concentration of timing risk	Recovery may take 10+ years (Japan 1990-2010, US 2000-2013); retiree may not survive the recovery.	Long bear markets

Critical observation: leverage improves the median terminal wealth (positive spread compounds) but worsens the worst-decile (P10) wealth by similar dollar magnitude - variance amplifies in both directions. Because the Safe Withdrawal Rate is constrained by the 10th-percentile path, **leverage typically reduces SWR even when it improves median terminal wealth.** This is the classic Kelly-criterion tension: leverage is a legacy-maximization tool, not a current-spending tool.

Statutory references: ITA s.20(1)(c) interest deductibility; *Singleton v. Canada* (2001 SCC 61) reasonable-expectation-of-income test; *Lipson v. Canada* (2009 SCC 1) GAAR limits on Smith Manoeuvre; ITA s.111 non-capital loss carry-forward.

10. Safe Withdrawal Rate Frontier

The Safe Withdrawal Rate (SWR) is the maximum constant real annual spending that keeps the probability of terminal-year wealth ≥ 0 above a user-specified confidence target. The model computes SWR across multiple confidence levels (50%, 70%, 80%, 85%, 90%, 95%, 99%) to display a frontier curve.

Algorithm

- Bracket phase: start at $\max(\text{input spend}, 3\% \text{ of wealth})$, expand by 1.4x until success drops below the target (max 6 expansions to find an upper bound).
- Contraction phase: if the first bracket already fails, contract by 0.6x until target met (finding a lower bound).
- Binary refinement: 7 iterations or until the bracket width is less than 0.1% of initial wealth.
- Per-step Monte Carlo: 1,500 simulations per binary-search probe (giving $\sim \pm 1\text{-}2$ percentage points of MC noise on the success rate).
- Success criterion: terminal-year wealth ≥ 0 , identical to the primary MC metric.

Interpretation

Two distinct metrics share the dashboard, and they answer different questions:

Question	Answer in the model	Where displayed
At my chosen spending, what fraction of paths succeed?	Success Rate % (e.g., 77.6%)	Success Rate metric tile
What spending level meets my 95% confidence target?	SWR (\$ amount, e.g., \$64,250/yr)	SWR banner + frontier chart

If the user's input spend is above the safe rate for their target confidence, the success rate at that spend will naturally be lower than the target - and the SWR section tells them what spend level *would* reach the target. The 95% confidence input is the target for the SWR computation, not a constraint that the simulator tries to satisfy at the user's current spend.

11. Validation Against Published Rates

The tax engine has been validated to basis-point precision against CRA, IRS, and major professional-services published rate tables (KPMG, EY, Deloitte tax facts publications). Selected validation points:

Rate / table	Validated value	Source
Top Ontario combined ordinary	53.53%	CRA / KPMG 2025 tax facts
Top Ontario capital gain effective	26.77% (50% × 53.53%)	ITA s.38(a) × marginal
Top Ontario eligible dividend	39.34%	CRA / EY 2025 tax facts
Top Ontario non-eligible dividend	47.74%	CRA / EY 2025 tax facts
IRS RMD divisors ages 73-120	All values match Treasury Reg s.1.401(a)(9)-9	IRS Pub. 590-B Appendix B
CRA RRIF factors ages 72-95+	All values match Income Tax Regs s.7308(4)	CRA published table
NIIT	3.8% on min(NII, MAGI - threshold)	I.R.C. s.1411
US top federal qualified dividend	20% LTCG + 3.8% NIIT = 23.8% federal	I.R.C. s.1(h)(11), s.1411
US top single LTCG + CA state	20% + 3.8% + 13.3% = 37.1%	I.R.C. + Cal. RTC
CCPC s.112 inter-corp dividend treatment	0% net (perfect integration)	ITA s.112
Holdco pipeline rate (Ontario)	26.77%	ITA s.70(5) deemed disposition × 50% × 53.53%

12. Antithesis and Known Limitations

Scientific honesty requires explicit acknowledgment of the model's simplifications. Where they matter for typical user inputs, the methodology section in the interactive tool flags them individually. Major limitations:

Tax model simplifications

- **Spouse-age election for RMD/RRIF:** Joint Life Table s.1.401(a)(9)-9(c) (US) and ITA s.146.3(1)(b.1) (CA) allow the use of a younger spouse's age, reducing mandatory withdrawals. Not modeled - single-life assumption produces conservative-upper-bound minimums.
- **Roth conversion ladder optimization** (US): not modeled as an optimization variable; could be added as future work.
- **Pension income splitting** (CA, ITA s.60.03): splitting up to 50% of eligible pension income with spouse - may reduce effective tax in couples. Not modeled.
- **OAS clawback threshold:** model uses \$90,997 (2024); the 2025 value is \$93,454 (off by ~\$2,500). Minor numerical correction.
- **75+ OAS bump:** the 10% OAS increase at age 75+ is implemented, but the proportionally-increased clawback ceiling for 75+ recipients is not separately modeled. Minor.
- **LIRA / LIF:** locked-in retirement accounts differ slightly from RRIF (annual maximum withdrawal) and are not separately modeled.
- **Inherited IRA SECURE Act 10-year rule:** out of scope; model assumes original owner.

Investment / portfolio simplifications

- **Single-asset return distribution:** mu and sigma describe the entire portfolio; no asset-class decomposition, no rebalancing modeling.
- **Real returns assumed:** mu and sigma are in real (post-inflation) terms; the inflation field is display-only. No fat tails (returns drawn from Gaussian); historical empirical distributions show heavier tails.
- **No tax-loss harvesting:** realized losses are not separately tracked and offset against gains. Real-world TLH can add 0.3-0.7% of after-tax return annually.
- **Cost basis stays constant for reinvested dividends:** US IRS treatment increases basis by reinvested-dividend amount; the model holds basis constant (simplification).
- **No transaction costs, ETF expense ratios, or advisor fees:** these are subtracted manually from the user's mu input.

Leverage simplifications

- **Loan balance constant** (interest-only, no amortization). Real loans often require principal repayment.
- **No margin-call forced liquidation:** in real life a 40%+ drawdown could trigger forced sale at the worst time, making P10 outcomes materially worse than simulated.
- **Real borrowing rate held constant** across the horizon. Variable-rate exposure not modeled.
- **Holdco-leverage mode applies interest deduction at personal level:** the proper mechanic would deduct at the corporate level against Holdco's investment income (reducing inside refundable Part I tax

and increasing RDTOH integration). Net effect approximately equivalent for typical retirees.

- **Holdco-leveraged funds still face the user-set inside drag:** in reality, if interest expense exactly matches investment income at the corp, inside drag on the leveraged portion would be zero. Model's drag is an aggregate-portfolio estimate not adjusted for leverage.

13. User Guide - Operating the Tool

Workflow

1. Select country (US / Canada) and filing status (US only).
2. Select state (US) or province (CA) for the second tax layer.
3. Enter current age, end age, and annual after-tax spending need.
4. Enter starting balances and cost basis (where applicable) for each bucket. Bucket types are country-specific but values carry across via equivalence mapping (Taxable ↔ Non-reg, etc.).
5. Set government pension amounts and start ages (SS for US; CPP + OAS for CA).
6. Set corporate inside-drag (% - for CCPC: corp tax on US dividends only).
7. Choose corporate extraction strategy (Pipeline / Dividend for CA; Salary / Dividend / Defer for US).
8. Set return assumptions (μ , σ , dividend yield, eligible % for CA).
9. (Optional) Set leverage parameters: % of portfolio, real borrowing rate, location (Personal / Holdco / Both).
10. Choose simulation parameters: number of sims (default 2000), confidence target (default 90%), bracket-fill ceiling (low/mid/auto).
11. Click Run. Wait ~3-30 seconds depending on parameters.
12. Review the dashboard: Success Rate, Lifetime Tax, Terminal Wealth distribution, SWR frontier, year-by-year schedule, and methodology.

Interpreting the dashboard

Success Rate (cyan tile): the probability that terminal-year wealth ≥ 0 at the user's chosen spend. If this is below the user's confidence target, the subtitle shows what spending level would meet the target (the SWR).

Lifetime Tax (red tile): median total personal tax across all simulation paths. This is the primary objective function - lower is better for the same level of spending.

Terminal Wealth - P50 (amber tile): median final wealth at end-age. P10 and P90 in subtitle give the dispersion. Includes Holdco pipeline tax conversion at terminal and loan repayment if leveraged.

Worst-Decile Floor (rose tile): the 10th-percentile terminal wealth - the value below which only 10% of simulated paths end. This is the relevant tail-risk number for SWR computation.

Wealth-fan chart: trajectories of P10, P25, P50, P75, P90 wealth over the horizon. Visualizes sequence-of-returns risk and corporate-bucket compounding.

SWR frontier: maximum sustainable spend at each confidence level (50% to 99%). The user's selected confidence level is highlighted; the dashed line shows their input spend.

Year-by-year schedule: representative-path table showing each year's withdrawals by bucket, pension income, tax, and net cash. The 'representative path' is the simulation whose lifetime tax is closest to the median.

Methodology section: dynamically rendered HTML explaining the calculation, statutory references, and antithesis caveats specific to the current configuration.

14. Statutory References

United States - Internal Revenue Code

- I.R.C. s.1(h) - Capital gains tax rates
- I.R.C. s.1(h)(11) - Qualified dividend treatment
- I.R.C. s.1(j) - 2017 Tax Cuts and Jobs Act bracket structure
- I.R.C. s.11 - Corporate income tax (21% federal)
- I.R.C. s.63(c) - Standard deduction
- I.R.C. s.86 - Social Security taxation (provisional-income method)
- I.R.C. s.162 - Trade or business deductions (reasonable compensation)
- I.R.C. s.163(d) - Investment interest expense deduction
- I.R.C. s.401(a)(9) - Required minimum distributions
- I.R.C. s.408A(c)(5) - Roth IRA: no RMD during owner's lifetime
- I.R.C. s.531-537 - Accumulated earnings tax (C-corp trap)
- I.R.C. s.541 - Personal holding company tax (C-corp trap)
- I.R.C. s.1014 - Step-up in basis at death
- I.R.C. s.1411 - Net Investment Income Tax (NIIT)
- I.R.C. s.3101(b)(2) - Additional Medicare Tax
- Treasury Reg s.1.401(a)(9)-9 - Uniform Lifetime Table (2022 update)
- SECURE Act 2.0 s.107 - RMD age moved to 73 (2023), 75 (2033)
- SECURE Act 2.0 s.325 - Roth 401(k) RMD elimination (2024)

Canada - Income Tax Act and Regulations

- ITA s.20(1)(c) - Interest deductibility on money borrowed for investment
- ITA s.21(2)-(3) Ontario - DTC applied after surtax
- ITA s.38(a) - Capital gain 50% inclusion
- ITA s.60.03 - Pension income splitting (not modeled)
- ITA s.70(5) - Deemed disposition at death
- ITA s.82(1)(a) - Non-eligible dividend gross-up (1.15x)
- ITA s.82(1)(b) - Eligible dividend gross-up (1.38x)
- ITA s.84(2) - Deemed dividend on wind-up (pipeline-attack provision)
- ITA s.84.1 - Anti-surplus-stripping rule (pipeline-compliance framework)
- ITA s.89(1) - Definition of 'eligible dividend' and GRIP
- ITA s.111 - Non-capital loss carry-forward (20 years)
- ITA s.112 - Inter-corporate dividend deduction
- ITA s.117 - Federal tax brackets

- ITA s.118(1) - Basic Personal Amount
- ITA s.118(2) - Age amount (65+)
- ITA s.121 - Federal Dividend Tax Credit (eligible 15.0198%, non-eligible 9.0301%)
- ITA s.123 - Corporate income tax
- ITA s.146.3 - RRIF rules; (b.1) spouse-age election (not modeled)
- ITA s.180.2 - OAS Recovery Tax (clawback)
- ITA s.245 - General Anti-Avoidance Rule (GAAR)
- Income Tax Regulations s.7308(3) - Early-conversion RRIF formula $1/(90 - \text{age})$
- Income Tax Regulations s.7308(4) - Post-71 RRIF prescribed factor table

Canadian case law

- Singleton v. Canada, 2001 SCC 61 - Reasonable expectation of income test for s.20(1)(c)
- Lipson v. Canada, 2009 SCC 1 - GAAR limits on Smith Manoeuvre
- MacDonald v. Canada, 2013 FCA 110 - s.84(2) deemed-dividend recharacterization
- Foix v. Canada, 2024 TCC - Extended GAAR scrutiny of pipeline variants
- Aspro Inc. v. Commissioner, T.C. Memo 2021-8 - Reasonable compensation in closely-held C-corp (US analog)

This document accompanies the interactive HTML tool. All tax computations are dynamically rendered in the tool with current parameter values. For questions, corrections, or methodology discussion, use the contact form on the project landing page.